Papers Arising from the Vertebral Artery Study

1. Brief Introduction to quaternions and framed vectors and their application to the description of anatomical movements.

The mathematical description of anatomical movements requires two types of objects: a means of symbolically expressing the location, extent, and orientation of an anatomical structure and a means of operating on those symbolic expressions as rotations and translations act upon the anatomical structure. The first need is fulfilled by particular arrays of vectors called framed vectors. The second need is met by quaternions.

A framed vector is a set of at least five vectors that express different attributes of the anatomical object. The first, location, vector expresses the position of the object relative to some reference point. The second, extension, vector represents some spatial attribute of the object, such as its longitudinal extent or the location of a muscle attachment relative to the object’s center. There may be multiple extension vectors. The last three vectors form a frame of reference. The frame of reference is a set of three non-coplanar vectors that express the orientation of the object. For example, the frame of reference for a head might have one vector directed anteriorly, one directed laterally, and one directed superiorly. As the head moves, the frame is carried with it and codifies its new orientation.

Quaternions are hypercomplex numbers consisting of a real number and three different imaginary numbers. The real number is a scalar and the three imaginary numbers form a vector. Quaternions are ideally suited to model rotations. A quaternion may be interpreted as the ratio of two vectors, therefore the vector in the denominator of the ratio multiplied by the quaternion is the vector in the numerator.

$$\boldsymbol{Q} = \frac{\mathbf{v_N}}{\mathbf{v_D}} \quad \Leftrightarrow \quad \boldsymbol{Q} * \mathbf{v_D} = \mathbf{v_N}$$

If an anatomical object is represented by a suitable framed vector and a rotation in space is represented by a quaternion, then the quaternion operating the framed vector will produce a new framed vector that accurately represents the new location, extension, and orientation of the anatomical object after that rotation. There are different rules for manner in which the attributes of an orientable object are transformed by rotation, translation, and re-scaling. These rules are embedded in the rules for how quaternions operate upon framed vectors. These basic rules are briefly introduced and a simple example is computed to illustrate the application of quaternion analysis to the description of anatomical movement.

It is shown that this approach utilizing quaternions and framed vectors provides an intuitive, unambiguous, symbolic and quantitative, language for describing anatomical movements. It is applied in the following papers to describe some problems revolving around the biomechanics of the neck.

2. The sensitivity of blood flow in the vertebral arteries to the biomechanics of the upper cervical spine.

Introduction: It has been observed that placing the head and neck in certain positions changes the flow of blood through the vertebral arteries. The most likely cause of the changes in blood flow are strains of the vertebral artery that change its shape and secondarily its resistance to flow. In this first step, it is determined which positions elicit changes in blood flow and what parameters are most affected.

Methods: In this paper the data from a series of 22 normal subjects is examined to determine what positions are most apt to reduce blood flow. Each subject was placed in a series of standard positions that are generally thought to stress the vertebral arteries. In each position, the blood flow at mid-cervical levels of each vertebral artery was measured with Duplex Doppler ultrasound. The measured parameters of blood flow were peak systolic velocity, end diastolic velocity. The resistive index was calculated.

The question posed was: What positions are associated with detectable increases or decreases in blood flow? This was addressed by looking at the relative numbers of tests that resulted in an increase or a decrease in flow, relative the blood flow in neutral position. The number of tests that produced a positive or negative change in flow was the statistic studied. Change in blood flow is a binary process and the probability of the relative numbers of positive and negative changes was computed, assuming a binomial process with equal probability of an increase or decrease of blood flow.

The number of tests in a particular position that showed marked reduction or cessation of blood flow was another statistic considered. The marked reduction or cessation of blood flow is considered an indication of substantial stress in the vertebral artery.

Results: Blood flow in neutral position is quite variable. This means that there is a substantial intrinsic noise level against which we are trying to detect what is usually a fairly small change. While there was a high noise level due to the normal variation in blood flow, it was possible to detect subtle changes in the blood flow.

Certain positions of the head and neck are significantly more likely to cause a decrease or increase in the parameters of blood flow. 1.) It was observed that contralateral lateral rotation is substantially more apt to reduce blood flow than ipsilateral lateral rotation or combinations of lateral rotation and extension. 2.) The position most effective in reducing blood flow was contralateral rotation in the premanipulative hold for manipulation of the C1/C2 joint. In that position the head and neck are sideflexed towards the artery and the atlas in rotated contralaterally upon the axis. 3.) Extension alone had no effect on blood flow. When added to lateral rotation, it tended to reduce the amount of change expected from lateral rotation alone.

Discussion: These observations are consistent with the fact that the vertebral arteries are strained far more in the interval between the transverse foraminae of the atlas and the axis and the principal movement in that joint is lateral rotation. They are also consistent with most strokes due to rapid head movements being due to injury in the C1/C2 segment of the vertebral artery.

The observed pattern of blood changes with respect to the test positions indicates that many of the so-called vertebral artery stress test positions are not stressful for the vertebral arteries. The lack of any conscious response to the complete cessation of blood in a vertebral artery, even when the opposite artery was half the diameter of the occluded artery, suggests that the vertebral artery stress tests will usually not provoke signs or symptoms even when there is a major compromise of blood flow. It remains to be proven if the provocation of signs and symptoms in individuals with positive stress tests is due to reduced blood flow.

They would seem to imply that there is more lateral rotation in the C1/C2 segment when the bones are placed in the premanipulative hold and less rotation when they are combined with extension. The modest increase in resistive index in the tests that combine extension and rotation suggest that there are other sites that may be affected by that combination.

3. The dynamics of blood flow in the vertebral artery: pulsatile versus steady flow

Introduction: Blood flow in the vertebral arteries has a characteristic waveform, that is modified by positioning the head and neck in certain endrange positions. In this paper, the question that is addressed is: how well do the two point measurements of blood velocity, peak systolic velocity and end diastolic velocity, correlate with the total velocity waveform and the flow volume through the arteries?

The observed changes in waveform are suggestive of changes in the pattern of blood flow. What can be deduced from the velocity waveform about the actual flow volumes and the resistances to flow in the posterior circulation to the brain?

Methods: Waveforms were analyzed and the relations between the measured parameters and blood flow were examined. The waveforms are numerically integrated to determine the amounts of flow in the baseline steady flow, the dynamic peaked flow and the entire pulse cycle. These measurements are compared to the values of two point estimates of the flow; the peak systolic velocity and the end diastolic velocity.

Results: It is found that though they are point measurements of extremes of the waveform, the peak systolic velocity was well correlated with total pulse flow, the end diastolic velocity was well correlated with steady flow, and the difference between the two measurements, called pulse height, was well correlated with the flow in the pulsatile part of the flow. Pulse height and end diastolic velocity were essentially uncorrelated.

It was also found that that there are simple relationships between the magnitudes of the peak systolic velocity and the end diastolic velocity and between the changes in their values in the test positions relative to their values in neutral position.

There was no correlation between the parameter values obtained in neutral position before testing and after testing and no tendency to either increase or decrease over the course of the testing.

Discussion: The two point measurements of the blood flow are good estimates of the total waveform's attributes and probably of the volume pulse flow.

The implications of changes in the peak systolic velocity, the end diastolic velocity, and the resistive index are examined. It is argued that baseline steady flow is an index of static resistance, pulsatile flow is an index of the dynamic properties of the system and resistive index reflects a shift in the balance between these two attributes of the system. The resistance to flow is similar to impedance in electrical circuits.

The lack of correlation between the parameters before and after testing indicates that the values during testing can be reliably referenced to the average flow in neutral position. It also argues that the variability in blood velocity is an intrinsic attribute of the system and it probably can not be reduced by varying the test paradigm.

4. A model of the biomechanical configuration of the upper cervical spine

Introduction: The upper cervical assembly or axio-atlanto-occipital assembly (AAOA) is a complex arrangement of specialized bones in the upper cervical spine and the occiput, that provide a gimbal-like interface between the lower cervical spine and the head. In this paper, the biomechanics of this array of bones and joints is modeled from the known anatomy of the region.

Methods: The bones of the AAOA and the axes of rotation for their joints are represented by framed vectors, which are modified by quaternions, representing the rotations themselves. These mathematical objects are incorporated in a program, written in Mathematica, that allows one to specify the angular excursion from neutral position in each joint and it returns the three-dimensional configuration of the bones in the AAOA.

Results: The composition of a set of framed vectors for the bony elements are described and typical values assigned to their elements. A set of rotation quaternions are chosen and the calculation of the configuration of the system is illustrated with a sample calculation.

A sample exercise is sketched that addresses the gap bridged by the vertebral arteries as they pass from their penetration of the posterior atlanto-occipital membrane to their fusion, to form the basilar artery. It is shown that the greatest potential for stressing the arteries is in endrange extension, but the likelihood of doing so is small.

Discussion: The model provides an opportunity to explicitly review and codify the known anatomy of the region. The model is such that it easy to change the parameters in light of new or alternate evidence on the anatomy. Having created the model, it is generally straightforward to ask biomechanical questions and obtain quantitative answers very rapidly. The model serves as a component of a number of other models that address particular questions about the biomechanics of the AAOA and the vertebral arteries.

5. A model of the biomechanical configuration of the lower cervical spine

Introduction: The lower cervical spine (C3 to T1) is a linked chain of similar units that provide most of the mobility between the thorax and the head. The types of movements allowed between the units are restricted and often oblique to the conventional anatomical orientation of the vertebrae. These take two forms; first, sagittal rotations about transverse axes, and, second, oblique rotations about axes that pass diagonally from posterior and superior to inferior and anterior, between the facets. In this paper the implications of this arrangement are explored by manipulating a mathematical model, constructed in Mathematica.

Methods: The lower cervical spine is represented by a set of similar units represented by framed vectors and the movements between the units are expressed as rotation quaternions. These features are expressed as a Mathematica program that allows one to enter the values for the rotations in the various joints and it computes the three-dimensional configuration of the array of bony elements.

Results: The movements of the lower cervical spine are such that there is an essential conjoint rotation between lateral rotation and sideflexion. The degree of conjoint movement is a function of the relative extension and flexion between the elements.

Because we are dealing with a chain of bony elements, the effects of off-center rotations are magnified by the cumulative rotations of several elements in the chain.

Discussion: Because of the interplay between of the roll and turn components of the lower cervical spine, there is an essential tilting of the axis vertebra for any movements that are not strictly sagittal. This is a reason that the AAOA is necessary, to act as a gimbal between the lower cervical spine and the head.

The relationship between the two types of rotation are explored and addressed in general terms. It is argued that the definitions of the movements are a function of the definitions of the orientations for the vertebrae.

6. Strain in the vertebral artery with lateral rotation in the atlanto-axial joint

Introduction: The AAOA model is used to compute the relative movements between the transverse foraminae of C1 and C2, the movements that strain the vertebral artery in that region. It is assumed that the vertebral artery acts like an elastic tube. The distortions in such a tube when it is twisted as by the rotation of the atlas about the dens are computed.

Methods: The biomechanical model of the AAOA is used to compute the movements of the transverse foraminae as the atlas rotates laterally about the odontoid process. This calculation is used as the basis for determining the boundary conditions straining the vertebral artery. The atlas and axis are rotated through varying degrees of lateral rotation and the strains imposed upon an elastic tube fixed at it ends in the transverse foraminae are computed, on the assumption that the tube remains circular in the foraminae or that it is allowed to become crimped.

Results: It is found that there are complex changes in the shape of the tube when the atlas is rotated about the odontoid process of the axis, causing a twisting or wringing motion in the vertebral arteries. These include elongation, flattening, pinching, and torsion. With crimping at the ends, there is more distortion in ways that depend upon which end is crimped and how much.

Discussion: Using the biomechanical model of the AAOA and a few simple assumptions about the elasticity of the vertebral artery, it is possible to determine how the vertebral artery is apt to be distorted by lateral rotation in the atlanto-axial joint. These calculations serve as the basis of calculations on the probable effects of strain in the vertebral artery upon blood flow through the artery.

7. Flow changes associated with strain in the vertebral artery due to lateral rotation in the atlanto-axial joint

Introduction: It is possible to compute the changes in blood flow that one would expect due to the strains in the vertebral artery as a result of lateral rotation in the atlanto-axial joint. These calculations give some idea of which factors are most apt to compromise blood flow and provide a basis for the observations of changes of blood flow when the head and neck of normal subjects are placed in various stress test positions.

Methods: programs were written in Mathematica to compute the flow through elliptical cross-sections of arbitrary dimensions. These functions are used to compute the distributions of flow parameters for laminar flow in strained elastic tubes that have been twisted as by rotation of the atlas about the dens of the axis.

Results: It is found that the strain of the vertebral artery produces substantial reduction in the flow through a twisted tube, producing an order of magnitude reduction in flow volume with endrange lateral rotations in the C1/C2 segment of the cervical spine. It is further observed that crimping will produce very sharp increases in the resistance to flow as one approaches endrange lateral rotation, possibly as much as two orders of magnitude.

Discussion: These calculations indicate that there are marked decreases in flow rate as one approaches endrange lateral rotation in either direction, but more so with contralateral rotation.

Crimping is the most effective means of restricting flow and it is probably responsible for the cessation of flow that is seen in some test positions.

8. The role of the alar ligament in the biomechanics of the upper cervical spine.

Introduction: The dramatic increase in the numbers of tests that cause cessation of blood flow in the vertebral arteries with contralateral rotation in the premanipulative hold for manipulation at the C1/C2 segment is probably due to an increase in the magnitude of the lateral rotation in the atlanto-axial joint. The two biomechanical conditions that exist in that test position are the neck and the head are side flexed towards the side of the affected artery and the atlas is rotated contralaterally upon the axis. It is speculated that the mechanism for this increase in lateral rotation involves the alar ligaments. This possibility is examined by computing the changes in the gaps that the alar ligaments traverse as the bony elements of the AAOA are rotated in the various joints.

Methods: The biomechanical model of the AAOA is used to compute the changes in the gaps spanned by the alar ligament as the occiput is rotated in the atlanto-occipital joint and the atlas is rotated upon the axis in the atlanto-axial joint.

Results: It is found that the alar ligaments are central players in both joints and that they mediate a complex interplay between the movements in the two joints. In particular, sideflexion in the atlanto-occipital joint removes the alar ligament as a restraint upon contralateral lateral rotation in the atlanto-axial joint.

Flexion or extension in the atlanto-occipital joint restricts the amount of lateral rotation in the atlanto-axial joint. When starting from central alignment, the alar ligaments do not restrict ipsilateral rotation.

Discussion: The dynamics of the alar ligaments are consistent with the observations of changes in blood flow in the vertebral artery stress tests. Lateral rotation is the movement that most strains the vertebral artery. The effects of the rotation increase steeply as one approaches the endranges. Sideflexion of the occiput releases the restraint that the alar ligament places upon contralateral lateral rotation, allowing greater lateral rotation and increasing the probability of cessation of blood flow. On the other hand, extension, tightens the alar ligaments, reducing the amount of lateral rotation in both directions, thereby reducing the probability of reduction of blood flow. Since the alar ligaments do not restrict ipsilateral rotation when the occiput is moving from central alignment, it is easier to rotate the atlas far enough ipsilaterally to compromise blood flow.